

GeoGebra 4

Calculus - Differentiation

Approximating the gradient of a curve with a chord

Creating Slider-dependent points on a graph

Open a new file and create 3 Slider controls named n , p and h . From the Slider Properties allow

- n to vary between -5 and +5 with a step-size increment of 1
- p to vary between -10 and +10 with a step-size increment of 0.1
- h to vary between +0.01 and +2 with a step-size increment of 0.01

Type the following equation into the Input Bar and press the <RETURN> or <ENTER> key.

$$f(x) = x^n$$


In the Input Bar, type $P = (p, f(p))$ and press the <RETURN> or <ENTER> key. This should put a point, P , on the curve $y = f(x)$ that is moveable when Slider p is moved.

In the Input Bar, type $Q = (p+h, f(p+h))$ and press the <RETURN> or <ENTER> key. This should put a second point, Q , on the curve $y = f(x)$ that remains a fixed horizontal distance, h , from point P .

Adjust Sliders p and h and confirm their relationship. Adjust Slider n to see how the curve alters.

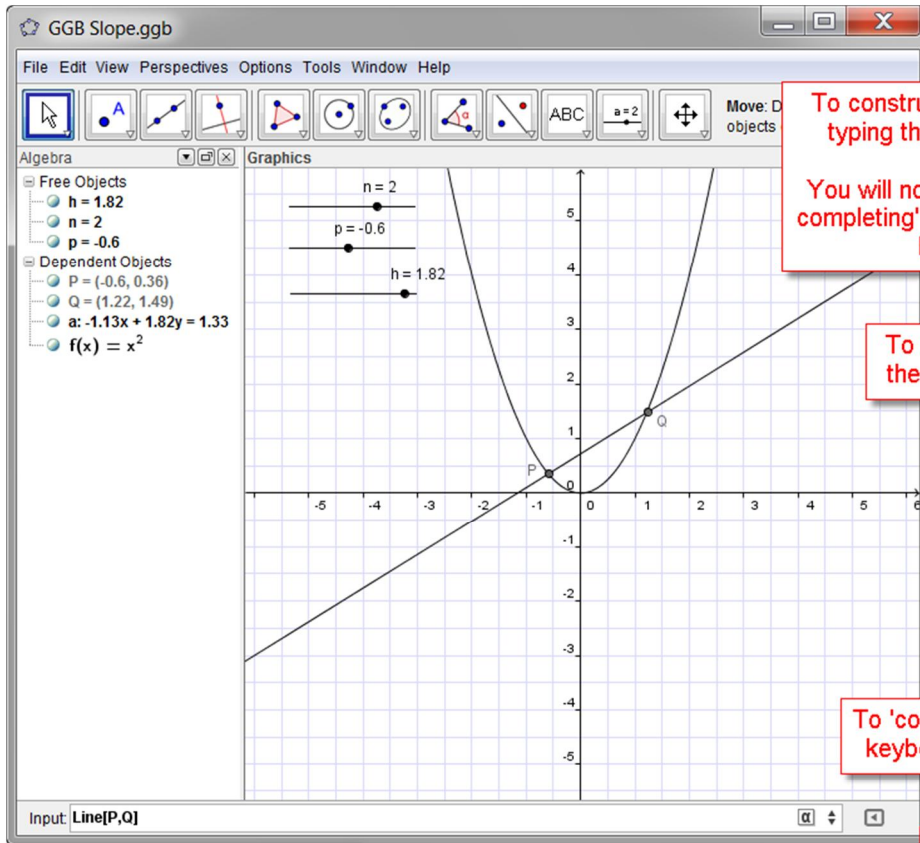
Drawing a line between two points

The Slow Method – not that slow really!

- If the “Line through two points” Icon  is visible as the face on the third Icon set on the Toolbar then click on it to select it. If another Tool is visible on this Icon then click on the white/red triangle and select the “Line through two points” option from the resulting drop-down Tool menu.
- Next, left mouse click on the point P , release the mouse button and then move to the point Q (you should now see the line being formed, fixed through point P). Finally, left mouse click on point Q to create and fix the line through P and Q .

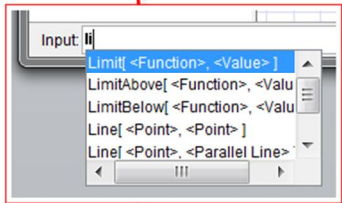
The Fast Method – *not much faster. HOWEVER, all GeoGebra functions are accessible via the Input Bar BUT not all have Icons associated with them.*

- In the Input Bar type Line[P, Q] and press the <RETURN> or <ENTER> key. See the diagram below to understand the time-saving that can be made with GeoGebra's Auto-fill capabilities for functions typed into its Input Bar.

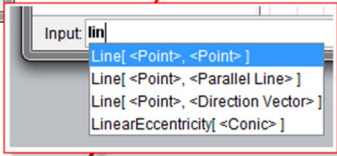


To construct a line between points P and Q begin typing the text: "Line[P, Q]" into the Input Line.
 You will notice that GeoGebra is capable of 'Auto-completing' text in the Input Bar, as is demonstrated below in the following images.

To move up or down the drop-down list use the Keyboard Up and Down Arrow Buttons.



To 'complete' a highlighted function press the keyboard <ENTER> or <RETURN> Button



The first argument to the function will automatically be highlighted after pressing <ENTER> or <RETURN>. If it is highlighted just type "P", there is no need to click on it first.

Press <ENTER> or <RETURN>




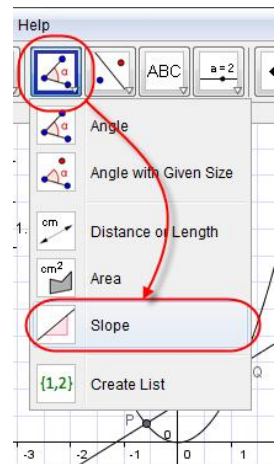
Press the <TAB> Button to move to the next argument in the 'Line' function. Type "Q".



Drawing a Slope Triangle between two points

The Slow Method – not that slow really!

- If the "Slope" icon  is visible as the face on the eighth icon set on the Toolbar then click on it to select it. If another Tool (most likely the Angle Tool) is visible on this icon then click on the white/red triangle and select the "Slope" option from the resulting drop-down Tool menu.



- Next, left mouse click on the line PQ . You will now see a small triangle created with its left vertex fixed to point P (Note that the Slope triangle created has, and maintains, a base width of one unit on the horizontal scale).

Definition

The tangent of the angle of the triangle at P is how the Slope or Gradient is defined. For example, an angle of 45° has a Slope or Gradient value of $\tan(45^\circ)$ or 1, smaller angles have Slope values less than 1 and larger angles have larger Slope values than 1.

The Fast Method

- In the Input Bar type "Slope[a]"

Now experiment with different Slider values of p , h and n . What observations can you make? Why was the lowest value of Slider h set to 0.01?

A note on naming objects

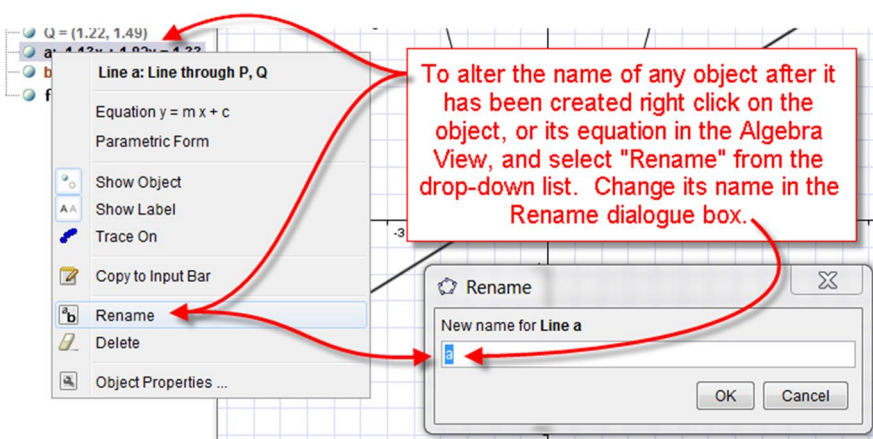
Names are most easily given to objects at their time of creation in the Input Bar.

Some examples of object having names of my choice include:

- **LineA = Line[P,Q]** would construct a line through points P and Q and name the line "LineA"
- **Grad_P = Slope[a]** would construct a Slope Triangle at point P on the line 'a' called "Grad_p"
- **Slope_{PQ} = Slope[a]** would construct a Slope Triangle at point P on the line 'a' called "Slope_{PQ}"

Notice that there cannot be any gaps in object names.

To give names to objects previously un-named at their time of creation – or to re-name them – follow the instructions given in the picture below:

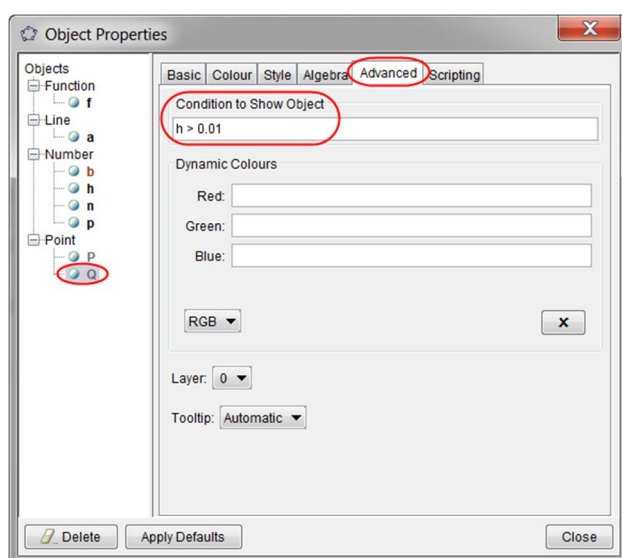


Conditional visibility

You may have noticed when you gave Slider h its lowest value above that the point Q appears to lie on top of point P with both labels visible. This is not quite true – it is 0.01 units to the right of P 's position. However, when the value of the Slider p is altered the separation between P and Q does become a little more visible if point P lies on a steep part of the curve. This can make the two, almost coalesced points and their labelling look somewhat “messy”.

The procedure below will make point Q vanish, together with its label, whenever $h = 0.01$.

- Right-mouse click on the point Q and select Object Properties from the drop-down list
- Under the “Advanced” Tab in the “Condition to show object” Text Input Box, type $h > 0.01$, as shown below:



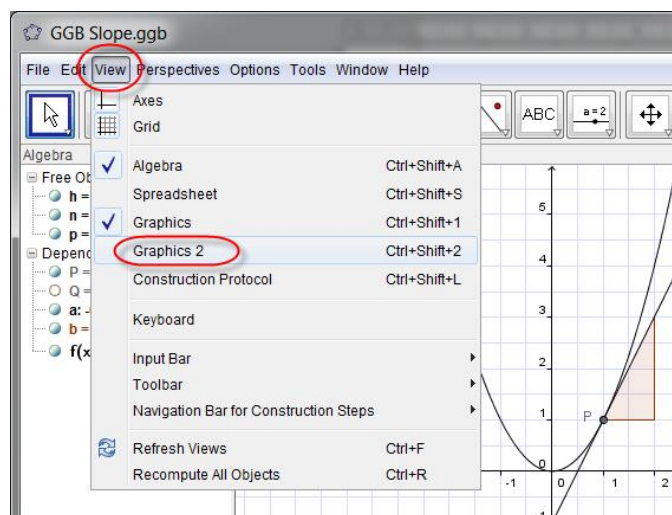
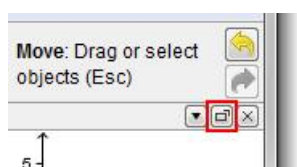
Your construction should now have greater clarity when h takes its minimum value.

Creating a 2nd Graphics Window

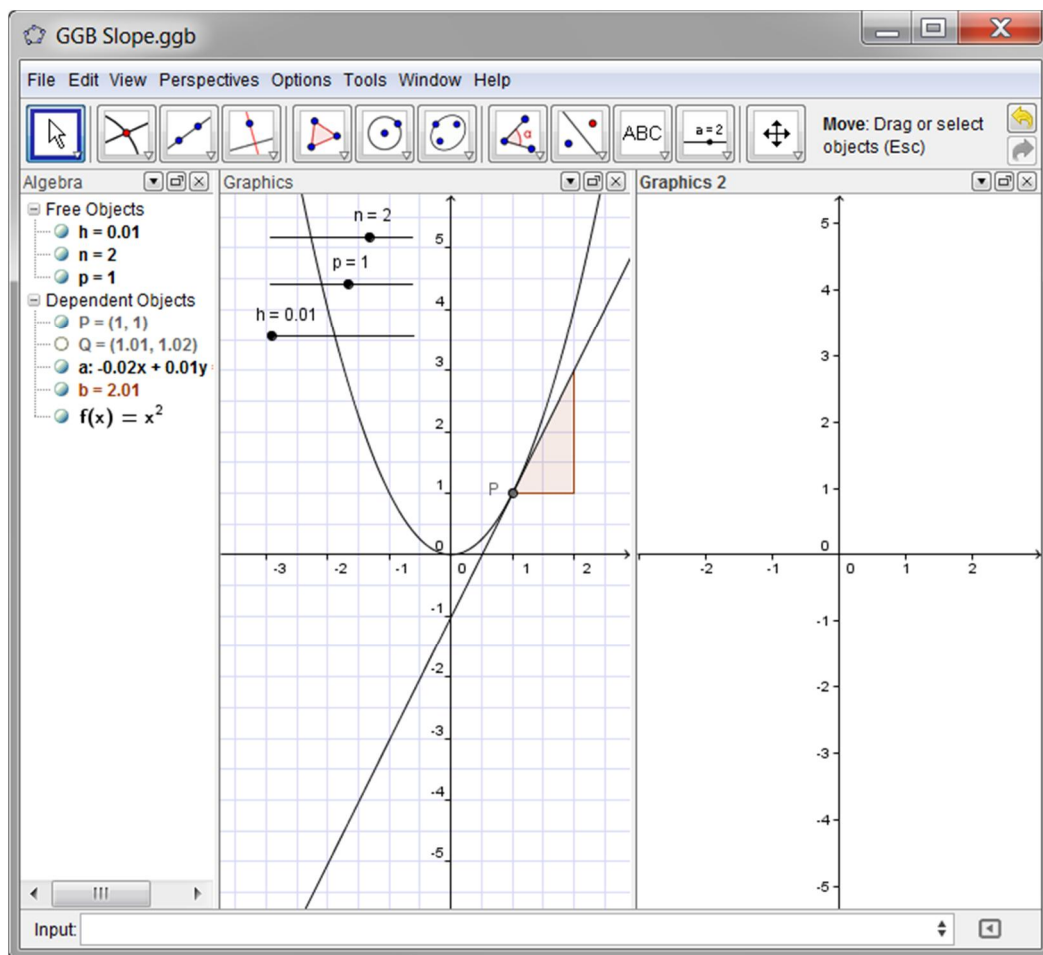
(In this case having a connection to the original Graphics 1 window)

From the View Menu select the “Graphics 2” option. This will now open up a 2nd Graphics window.

This screen can be “docked” together with the other windows or it can be “free-floating”. At the top of each window are three small icons, the middle of which controls the “docked” or “free” status of each window when clicked repeatedly.



When “docked” and resized your window may look something like this screenshot below:



Notice that there remains only one Input Bar and so, depending upon which Graphics window we wish functions inputted from the Input Bar to act in, we must first activate the required Graphics window, 1 or 2, by clicking anywhere on it before entering our function or formulae into the Input Bar.

Creating a locus

What we will now do is to plot the locus of point having its x-coordinate the same value as in the previously drawn graph in the Graphics 1 Window and its y-coordinate as the value of the slope from the same graph at the given x-coordinate.

First we need to create the point having these x and y-coordinate definitions but, in this example, we will construct it in our Graphics 2 window.

- Left-mouse click anywhere in the Graphics 2 window to activate it.
- In the Input Bar type the following:

$$P' = (x(P), b)$$

where

- $x(P)$ is the x-coordinate of the graph in the Graphics 1 window, and
- b is the value of the slope or gradient of the graph in the Graphics 1 window at the point $x(P)$ – as automatically labelled by GeoGebra within the Algebra View window.

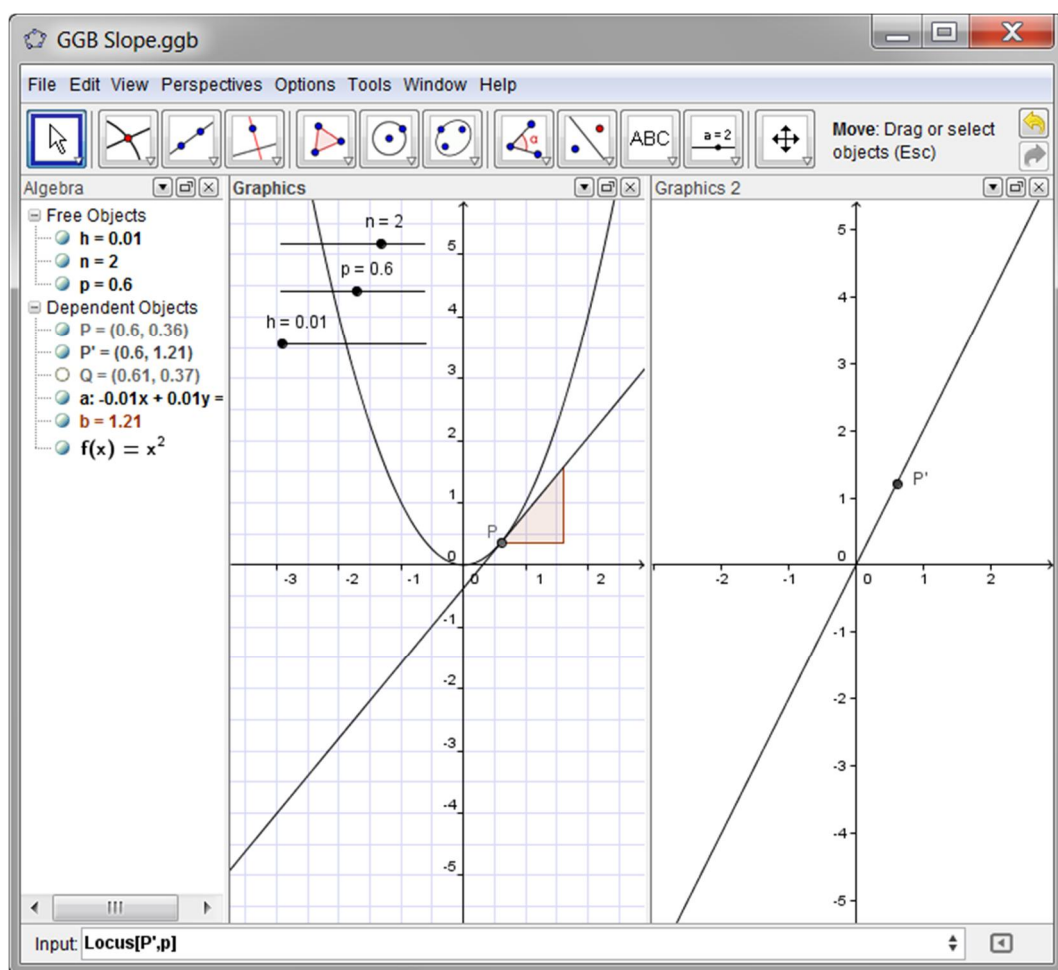
To construct the locus of this new point when the point P is moved in the Graphics 1 window first make sure that the Graphics 2 window has been activated by clicking in it, then type the following in to the Input Bar:

$$\text{Locus}[P',p]$$

and press <RETURN> or <ENTER>.

You will notice from the drop-down, auto-complete list, that this is the 2nd form of Locus function that GeoGebra is capable of generating, one that depends upon a Slider value; namely our p -Slider that controls the x-coordinate of the original point P in the Graphics 1 window.

If $n = 2$ in your Graphics 1 window then your file may look like this below:



Try altering your Slider values now and list below your observations about the function graph and its associated Slope or Gradient graph.

You may find it instructive to right-click on the function in the Algebra window and modify it to take the form:

$$f(x) = \frac{x^n}{n}$$

Save your GeoGebra file as "Gradient curve.ggb"

Observations...

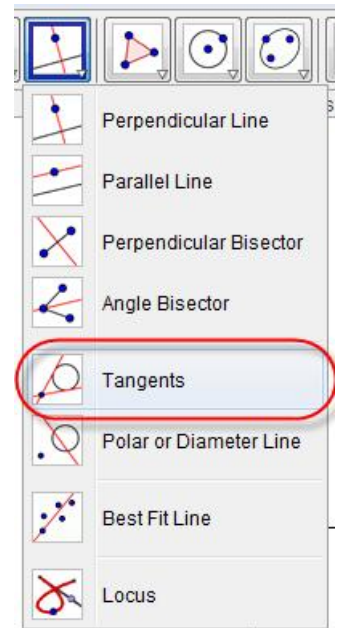
Creating a Tangent

Unsurprisingly, GeoGebra has within it a tool to automatically construct a tangent to a function curve at a point on the curve.

Method 1 – Using a Toolbar Icon

Select the “Tangents” Icon from the 4th Icon set on the Toolbar and with this tool selected first click on the **point** on the curve at which you want the tangent to be constructed, then click on the **curve** to complete the steps required for automatic construction of the tangent(s).

Note that the point need not be on the curve, and if this is the case then tangent(s) will be constructed **from** the point **to** the curve, rather than **at** the point **on** the curve, if possible.



Method 2 – Using the Input Bar

As you begin typing “tangent...” in the Input Bar you will see five available options within GeoGebra for it to construct tangents.

For the simple case of a tangent **to** a curve at a point **on** the curve you need to select the option:

$$\text{Tangent}[\langle \text{Point on Curve} \rangle, \langle \text{Curve} \rangle]$$

and use the names that either you have given, or GeoGebra has automatically given, the “Point on Curve” and “Curve” objects (Look in the Algebra view window to see names)

New tangent project

Open a new GeoGebra file

- Create a new function curve in the Graphics 1 window, give it the name “MyGraph”. This function can take any form and could depend upon sliders if you wish.
- Put a free point, give it the name “P”, anywhere on your function graph – you should be able to drag this point P along the graph
- Construct an exact tangent to your graph at this movable point P, give it the name “MyTangent”
- Open the Graphics 2 window and activate it
- Create a point P' in the Graphics 2 window having the form:

$$(x(P), \text{Slope}[\text{MyTangent}])$$

- Construct the locus of this point P' in Graphics 2 window as P moves around your function curve in Graphics 1 window. Type: Locus[P',P] in the Input Bar.
- Drag point P. What do you observe?

